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SEMINÁRIO BRASILEIRO

DE

ANÁLISE

SEPARATAS

1997

A UNSTABLE WEAK SOLUTION TO A FREE BOUNDARY PROBLEM IN DIFFERENTIAL GEOMETRY

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ABSTRACT. In this article we prove the existence of a unstable solution of class $L^{1,2}$ to a Free Boundary Problem related to the Harmonic Map Equation defined on a Riemannian Manifold of dimension 2, and which target space is a Riemannian Manifold. The method used relies on the fact that the Minimax principle can be applied to the α-energy functional introduced by Sacks-Uhlenbeck.

1. INTRODUCTION

Let (N,g) be a n-dimensional Riemannian Manifold and $S \subset M$ a closed submanifold of M. Given a Riemannian Manifold (M,γ) of dimension 2, which boundary is non-empty, we may ask if there is a harmonic map $\phi : (M,\gamma) \to (N,g)$ such that
1.1 - $\phi(\partial M) \subset S$.
1.2 - $\partial_n\phi(w) \perp T_{\phi(w)}S$ for all $w \in \partial M$ ($\partial_n\phi = d\phi.n$ is the derivative in the normal direction to ∂M induced by the orientation on M).

In the case of dim(M)=2 the problem is more approachable since the harmonic map equation has order 2 and in this case the dimension 2 of the domain is exactly the critical dimension for the Sobolev Embeddings.

We are able to prove the existence of an unstable $L^{1,2}$-solution in Theorem 4.1 using the Minimax Principle applied to the α-energy functional first introduceded by Sacks-Uhlenbeck [2]. A harder problem is to study the C^∞ regularity of this weak solution. This study will appear in a forthcoming paper; for while we only managed to prove it in the case of $M = D^2$. In the case N = R^k and M = D^2, the result was first proved in Struwe [1].

2. BRIEF RIEMANNIAN GEOMETRY

A Riemannian metric γ defined on M allow one to introduce the Covariant derivative concept through the unique Levi-Civita Connexion. It means that given γ, we have a unique linear map $\nabla : \Gamma(TM) \to \Gamma(T^*M \otimes TN)$ satisfying the Leibniz rule $\nabla(f.s) = df \otimes s + f.\nabla s$, for all f$\in C^\infty$ and s$\in \Gamma(TM)$, where TN = tangent bundle of M, T^*M = cotangent bundle of M and Γ(E) = space of sections of E, whenever E is a vector bundle. The same construction is done over N.

A differential map $f : M \to N$ defines a section df$\in \Gamma(T^*M \otimes TN)$, where df_x is the derivative of f at $x \in M$. Once the vector bundle $T^*M \otimes TN$ is a Riemannian bundle, we define

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Date: 08/04/1997.
1991 ***Mathematics Subject Classification.*** Global Analysis.
Key words and phrases. Harmonic Maps, Minimax Principle, Riemannian Manifolds.
To the Organisers of the 45ºSeminário Brasileiro de Análise realised in May/97, Florianópolis - SC, Brazil.