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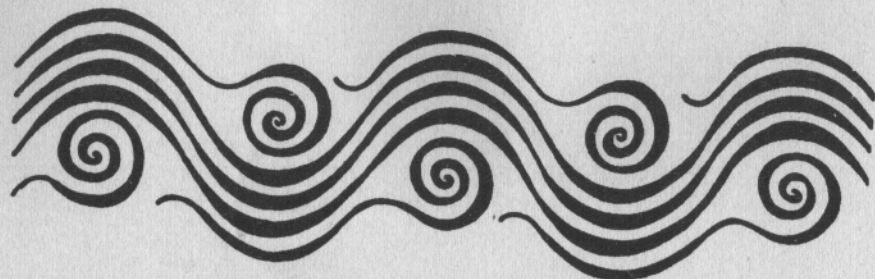
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A UNSTABLE WEAK SOLUTION TO A FREE BOUNDARY PROBLEM IN DIFFERENTIAL GEOMETRY

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ABSTRACT. In this article we prove the existence of an unstable solution of class $L^{1,2}$ to a Free Boundary Problem related to the Harmonic Map Equation defined on a Riemannian Manifold of dimension 2, and which target space is a Riemannian Manifold. The method used relies on the fact that the Minimax principle can be applied to the α -energy functional introduced by Sacks-Uhlenbeck.

1. INTRODUCTION

Let (N, g) be a n -dimensional Riemannian Manifold and $S \subset M$ a closed submanifold of M . Given a Riemannian Manifold (M, γ) of dimension 2, whose boundary is non-empty, we may ask if there is a harmonic map $\phi : (M, \gamma) \rightarrow (N, g)$ such that

1.1 - $\phi(\partial M) \subset S$.

1.2 - $\partial_n \phi(w) \perp T_{\phi(w)} S$ for all $w \in \partial M$ ($\partial_n \phi = d\phi.n$ is the derivative in the normal direction to ∂M induced by the orientation on M).

In the case of $\dim(M)=2$ the problem is more approachable since the harmonic map equation has order 2 and in this case the dimension 2 of the domain is exactly the critical dimension for the Sobolev Embeddings.

We are able to prove the existence of an unstable $L^{1,2}$ -solution in Theorem 4.1 using the Minimax Principle applied to the α -energy functional first introduced by Sacks-Uhlenbeck [2]. A harder problem is to study the C^∞ regularity of this weak solution. This study will appear in a forthcoming paper; for while we only managed to prove it in the case of $M = D^2$. In the case $N = R^k$ and $M = D^2$, the result was first proved in Struwe [1].

2. BRIEF RIEMANNIAN GEOMETRY

A Riemannian metric γ defined on M allow one to introduce the Covariant derivative concept through the unique Levi-Civita Connexion. It means that given γ , we have a unique linear map $\nabla : \Gamma(TM) \rightarrow \Gamma(T^*M \otimes TN)$ satisfying the Leibniz rule $\nabla(f.s) = df \otimes s + f.\nabla s$, for all $f \in C^\infty$ and $s \in \Gamma(TM)$, where TN = tangent bundle of M , T^*M = cotangent bundle of M and $\Gamma(E)$ = space of sections of E , whenever E is a vector bundle. The same construction is done over N .

A differential map $f : M \rightarrow N$ defines a section $df \in \Gamma(T^*M \otimes TN)$, where df_x is the derivative of f at $x \in M$. Once the vector bundle $T^*M \otimes TN$ is a Riemannian bundle, we define

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